Callendar-Van Dusen equations for the calibration of platinum resistance thermometers

WIKA data sheet IN 00.29

The accuracy of a platinum resistance thermometer (PRT) can be improved by the calculation of coefficients.

PRTs are usually calibrated at several temperature points in a temperature range requested by the user. Generally, the thermometer is not used at the exact calibration points, but also in between. This is why the user ofter requests a continuous description of the correlation between temperature and resistance over the complete temperature range used.

In most cases, this is achieved by the specification of a mathematical equation that describes the measured temperature points as an approximation.

Also widely used is the Callendar-van Dusen equation (CvD) which is used in DIN EN IEC 60751 as well in order to illustrate the so-called DIN characteristic curve.

In case of moderate requirements for measurement uncertainty, it is suitable for common Pt100 models across a wide temperature range.

The coefficients A, B, C and the conversion into α, δ, β

The relationship between resistance and temperature for platinum RTDs can be described by a polynomial.

In the early days of thermometry, Hugh Longbourne Callendar (1863 - 1930), British physicist, used a simple quadratic equation. Milton S. van Dusen, American chemist, later found that a third order term was required to adequately describe the relationship for temperatures below 0 °C. This gave rise to the Callendar-van Dusen equations still valid today:

For t > 0 °C:

$$R_t = R_0 \, (1 + At + Bt^2)$$

For t < 0 °C:

$$R_t = R_0 \, (1 + At + Bt^2 + C\,(t - 100)\,t^3)$$

Legend:
t = temperature in °C
R_t = resistance at temperature t
R_0 = resistance at 0 °C

These equations were used to establish the international temperature scale of 1927 (ITS-27) between 1927 and 1990. Since 1990 a more sophisticated equation has been used at national standards level (as described in the international temperature scale of 1990 (ITS-90)), but the Callendar-van Dusen equations are still widely used with industrial PRTs.

Historically, the equations were written in an alternative but equivalent form:

For t > 0 °C:

$$R_t = R_0 \left\{ 1 + \alpha \left[t + \delta \frac{t}{100} \left(1 - \frac{t}{100} \right) \right] \right\}$$

For t < 0 °C:

$$R_t = R_0 \left\{ 1 + \alpha \left[t + \delta \frac{t}{100} \left(1 - \frac{t}{100} \right) + \beta \left(\frac{t}{100} \right)^3 \left(1 - \frac{t}{100} \right) \right] \right\}$$

Although this looks more complex than the version using A, B and C coefficients, it is easier to derive the coefficients from calibration data. This form was therefore favoured before calculators and computers were available. It is still often used today, especially in the USA.

Similar technical information:
Usage limitations and accuracies of platinum resistance thermometers per EN 60751: 2008; see IN 00.17

These forms of the equation are equivalent and it is a simple matter to convert the coefficients from one form into the other:

$$A = \alpha \left\{ 1 + \frac{\delta}{100} \right\}$$

$$B = \frac{-\alpha\delta}{10^4}$$

$$C = \frac{-\alpha\beta}{10^8}$$

$$\alpha = A + 100B$$

$$\delta = \frac{-10^4 \cdot B}{(A + 100B)} = \frac{-10^4 \cdot B}{\alpha}$$

$$\beta = \frac{-10^8 \cdot C}{(A + 100B)} = \frac{-10^8 \cdot C}{\alpha}$$

For best accuracy, a PRT should be individually calibrated to generate the A, B, C or α, δ, β coefficients.

Alternatively, for less accurate temperature measurement generic values can be used. With generic coefficients, the accuracy of the temperature measurement depends on a number of factors, the most important being the purity of the platinum.

The purity of the platinum is indicated by the α value, which is easily determined as the average slope of the line between the ice and steam points on the resistance-temperature curve:

$$\alpha = \frac{R_{100\,°C} - R_{0\,°C}}{100 \cdot R_{0\,°C}}$$

Typically, industrial PRTs have a nominal alpha value of $\alpha = 3.85 \cdot 10^{-3}$ per °C.
For this grade of PRT, standard EN 60751:1995 provides values for the coefficients of:

A = $3.9083 \cdot 10^{-3}$ °C^{-1}
B = $-5.775 \cdot 10^{-7}$ °C^{-2}
C = $-4.183 \cdot 10^{-12}$ °C^{-4}

The converted values are as follows:
$\alpha = 3.85 \cdot 10^{-3}$ °C^{-1}
$\delta = 1.500$ °C
$\beta = 0.1086$

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